Energy in Wave Propagation :-

Potential energy : the pot. energy is given by

$$dU = -F(x)\,dx \quad \text{——— (1)}$$

Replace $F(x) = -m\omega^2 x$ in eq. (1) we get

$$dU = m\omega^2 x\,dx \quad \text{——— (2)}$$

Integrating eq. (2) from 0 to x, we get

~~$dU = \frac{1}{2}$~~

$$U = m\omega^2 \int_0^x x\,dx = \frac{1}{2} m\omega^2 x^2$$

or, $U = \frac{1}{2} Kx^2 = \frac{1}{2} KA^2 \sin^2(\omega t + \delta)$ ——— (3)

Let us also calculate the average PE of the system over one complete cycle, then it is calculate by integrating it over time from 0 to T (one time period).

$$U = \frac{1}{2} KA^2 \left[\frac{\int_0^T \sin(\omega t + \delta)\,dt}{\int_0^T dt} \right]$$

$$= \frac{1}{2} KA^2 \left[\frac{1}{2} \right]$$

$$\therefore U = \frac{1}{4} KA^2 \quad \text{——— (4)}$$

Kinetic Energy :- It is given by

$$K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \delta)$$

$$\therefore K = \frac{1}{2} KA^2 \cos^2(\omega t + \delta) \quad \text{——— (5)}$$

The average kinetic energy in one time period is given by

$$K = \frac{1}{2} KA^2 \left[\frac{\int_0^T \cos^2(\omega t + \delta)\,dt}{\int_0^T dt} \right]$$

$$= \frac{1}{2} KA^2 \left(\frac{1}{2} \right)$$

$$\therefore K = \frac{1}{4} KA^2 \quad \text{——— (6)}$$